Abstract

Snow avalanches are natural hazards which could severely damage infrastructures and thus pose a great threat to life in mountainous regions. This paper includes theoretical information on avalanche dynamics and modeling, as well as a practical application of the Voellmy-Salm avalanche model. The goal is to show whether the Voellmy-Salm model, which was originally created for modeling large-scale snow avalanches, can be applied to small-scale sand avalanches. This question is examined by comparing the runout distances of experimental sand flows to corresponding runout distances calculated by the Voellmy-Salm model. In the experiment, the sand slides down a self-built, wooden chute with four meters length, one meter width, and four different inclination angles. In order to calculate the runout distance, the values of the model's two friction coefficients ($\mu$ and $\xi$) need to be known. Since literature does not provide values applicable to this specific experiment, the values of $\mu$ and $\xi$ are found by calibrating the calculated runout distances to the experimental runout distances using try-and-error. The resulting values are $\mu = 0.29$ and $\xi = 255\text{ms}^{-2}$. With these values, the Voellmy-Salm model reproduces the experimental runout distances quite accurately. As a conclusion, it has been shown that the Voellmy-Salm model is capable of accurately calculating the expected runout distances of even small-scale sand flows, given that the appropriate friction coefficient values are used.
Acknowledgement

The final outcome of this project was only possible thanks to guidance and assistance from many people and I am extremely privileged and grateful to have received all this.

I especially thank Dr. Pál Molnár for guiding and supervising me until the completion of my project. I am grateful for his helpful advice and respect his dedication to add a finishing touch to my statistics.

My deepest gratitude goes out to Mr. Thierry Darbellay for introducing me to the Voellmy-Salm model. His expert knowledge was of fundamental importance for the formulation of my research question. The realization of this project would not have been possible without him.

I want to thank my father, Dr. Ronald Ruepp, very much for helping me build the chute. He was there for me all the way from the initial designing of the chute until the conduction of the experiment. This part of the project was by far the most enjoyable.

I heartily thank my mother, Yvonne Ruepp, and my sister, Marah Ruepp, for making possible each tedious round of experiments. Without their help and useful inputs, I would still be in the backyard scooping sand.

Last but not least, I am deeply grateful to Mrs. Dorothy Kohl for proof-reading the entire paper.
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1. Introduction

1.1 Motivation
With great respect snow avalanches are sometimes called “the white death”. Their frequent appearance poses a great threat to inhabitants of mountainous regions. Even the smallest of avalanches can bring death to careless skiers, and major ones can easily erase a whole village.

I am a very passionate skier. I am therefore often confronted with this violent and impressive phenomenon myself. Passing lots of time in the Alps every winter, I can see their impact on nature, population and infrastructure. Non-forested strips on mountainside bear witness to past but recurring avalanches. Nowadays skiers do not leave the safe ski slopes without avalanche transceivers and probes, and sometimes they even wear expensive avalanche airbag backpacks. Furthermore, avalanche barriers can be spotted on every critical hillside.

Avalanches have always fascinated me. Because I have encountered them in real life before. I am now eager to investigate this phenomenon from a different point of view, and hence dedicate this paper to their scientific investigation.

1.2 Research question
I will approach the following research question:

- Can the runout distance of small-sized sand avalanches be predicted using the snow avalanche Voellmy-Salm model?

This specific question will be examined using a self-designed experiment. In this experiment, I will let sand slide down a wooden chute, constructed by myself. More details are provided in section 5.

In the following theoretical part, the topics of avalanches in general and avalanche dynamics in particular will be introduced. In section 5, I will show how I came up with this very specific research question and how it can be examined with my experiment.
2. Avalanche research

2.1 Definition of snow avalanches

For the theoretical study, only snow avalanches (snowslides) will be considered. Avalanches are a large, rapid flow of snow down an inclined surface. They are released on hillsides and slide down toward the valley under the influence of gravity. However, this definition can be extended to further materials like debris (debris flows) or rock (rockslides) [HTL, p. 1]

2.2 Purpose of research [L1, L2]

I will start by explaining why avalanches are even studied. The first research of avalanches was conducted in the Swiss Alps by the Commission for Snow and Avalanche Research in 1931. This commission later became the SLF (Institute for Snow and Avalanche Research), situated in Davos, Switzerland. They are one of the leading institutes for avalanche research, worldwide. The need for reliable scientific methods to predict avalanche hazards, was due to both an expanding ski tourism in the early 20th century, as well as increased settlement in the Alps. Railway and hydropower plant companies also demanded more secure and reliable knowledge for constructing the first avalanche protection barriers and dams.

They all feared the destructive power of avalanches, regardless of size. The impact of so-called extreme avalanches became drastically apparent in the Alps during the winter of 1951, when whole buildings were buried, and 265 people died. Such avalanches fall in the category of natural catastrophes, and they are the main subject of research. Nevertheless, out of the 1500 avalanches which occurred in the Swiss Alps that winter, only 20 effectively caused damage in areas with avalanche protection measures. This proved the significant importance of research of avalanche dynamics, forecasting and protection. In the following years, over 1.6 billion Swiss Francs were invested into further research and construction of avalanche protection measures [L3].

2.3 Areas of research

Because the topic of avalanches is so extensive, I had to narrow down my area of research. Nevertheless, superficial knowledge for each area of research is important for
the understanding of the basic properties of avalanches. One differentiates between the following areas of study [L4]:

- **Snow physics [L5]:**
  The physical properties and the mechanical behavior of snow are examined. This proves to be difficult, since snow is a very complex and constantly changing material. Snow consists of water in all three aggregate states: solid (snow crystals), fluid (enclosed water) and gaseous (air and water vapor in the cavities). Depending on the ratio of these three states, snow can have very different physical properties, which strongly influences the dynamics of an avalanche. The water content plays an important role when classifying and calculating an avalanche (see section 3.2 and 4.3.2. Regarding material properties, snow counts as a viscous, porous, cohesive and granular medium that becomes brittle at high deformation speeds [HTL, p.32]. Researchers are especially interested in the microstructure and behavior of so called weak-layers. They are a requirement for the formation of avalanches (see below).

- **Avalanche formation and initiation [L6; HTL, p. 40f.]:**
  This field of study investigates the factors which lead to an instable snowpack, as well as the processes which take place before and during an avalanche initiation. The major goal is to assess snowpack stability in order to forecast imminent avalanches. A snowpack on an inclined plane is always subject to gravitational force. This leads to stress and strain [L7] within the snowpack. Tensile stress is built up in areas of increasing gradient (anticlines). Compressive stress occurs in areas of decreasing gradient (synclines). However, it is shear stress that is decisive for the formation of avalanches. It is built up because the bottom snow layer is fixed to the ground whilst layers above slightly slide towards the valley. This is known as creep movement. The larger the snowpack, the steeper the gradient, the weaker the connection between neighboring layers, and the closer to the snow surface, the bigger is the shear stress. When the force of shear stress exceeds the force by which the snow crystals are connected, the snowpack "breaks". This happens within the weak-layer, the layer in which the snow crystals have the weakest connectivity. This break can expand in all directions up to several hundred meters. A horizontal crack will become visible on the snow surface. The snowpack below
the crack is no longer connected to the snowpack above. If the gradient is at least 30°, the thrust exerted by the lower snowpack will now exceed its friction forces. As a result, the snow will start sliding downwards on the weak-layer and a so-called slab avalanche is formed. A slab is a cohesive layer of snow lying on a weaker one. This type of avalanche initiation is the most common and dangerous. It can be triggered spontaneously or by an additional force (e.g. weight of skier or avalanche blasting).

- **Avalanche Protection Measures [L8]:**
  In this area of research, protection measures are designed, constructed, and tested, often directly in the field. Their efficiency is assessed, both in safety and economy.

- **Snow climatology [L9]:**
  The interaction between snowpack and climate is investigated. Evaluating the change in density, depth, temperature, water content and distribution of snow in relation to certain climatic conditions is of great importance for avalanche forecasting.

- **Avalanche Dynamics and Risk Management [L10]:**
  This area of research investigates the avalanche movement from initiation on the hillside until the deposition and full stop in an area with a gradient below the critical angle. This is mostly found in the valley (explained later). Mathematical and physical models are developed in order to calculate certain characteristic values. Two of them are especially important for risk analysis:
  - run-out distance $s$ (maximum distance an avalanche moves into the valley)
  - impact pressure $p$ (pressure exerted by snow mass on object in avalanche track)

  When a potential avalanche track has been identified, these two values are used to assess whether it is safe to build houses or any infrastructure within or below it (Hazard zone mapping). Furthermore, the dimensions of avalanche protection measures such as avalanche dams or any reinforcement of house walls and windows can then be calculated.

This is the area of study I have chosen to focus on.
3. Physics of avalanches

3.1 Avalanche movement [L11; HTL, p. 66ff.]

After the snow cover is fractured and a slab is released, it will accelerate downwards. As this block gains speed, it breaks down into smaller fragments, which are constantly colliding, deforming, reconsolidating, or dividing further. When the particles become fine enough, the tumbling and sliding avalanche develops into a gravitational flow of continuous medium. This transformation is important for developing avalanche models (see section 4.3.1).

There are two main opposing forces acting on this gravitational mass movement. The accelerating force \( F_a \) is the component of the gravitational force \( F_g = m \cdot \vec{g} \) and acts parallel to the slope. Its formula is \( F_a = \sin(\varphi) \cdot F_g \) where \( \varphi \) stands for the slopes’ gradient.

The decelerating force \( F_r \) consists of various internal and external forces, which are explained in section 3.3.

We know from Newton’s second law of motion [PfSnE, p. 97] that an object accelerates when a net force \( F_{\text{net}} \neq 0 \) acts on an object. In the case of avalanches \( F_{\text{net}} = F_a + F_r \). The avalanche will accelerate downwards as long as \( F_a > F_r \). This condition is satisfied when the slab is released. This area is referred to as the starting zone and has a gradient of at least 30°.

As the avalanche gains speed, internal friction and hence \( F_r \) increases. When \( F_a = F_r \), a constant velocity is reached. The part of the avalanche path in which the snow flow either accelerates or moves at constant velocity is called avalanche track. In reality, the track velocity constantly changes in response to gradient, topography, vegetation, entrainment and deposition of snow. It’s a known fact that in the starting zone and track, the avalanche rather entrains more snow that it deposits.

Deceleration occurs when the gradient falls below the critical angle. \( F_a \) decreases because \( \varphi \) decreases and so \( F_a < F_r \). This area is known as the runout zone (usually below 20° gradient). Here, snow debris is deposited along the way and the avalanche comes to a full stop. The front marks the maximum runout distance.
This description of the external characteristics of avalanche movements applies to all avalanches regardless of size and type.

\[
\vec{F}_{\text{net}} = \vec{F}_a = \vec{F}_r = \vec{F}_g = \vec{F}_N
\]

Diagram 1: Basic forces acting on snow body (blue)

3.2 Avalanche types [HTL, p. 22f.]

The internal processes and their intensities vary between different types of avalanches. In order to understand these, the major avalanche types need to be known.

Even though avalanches in general can be morphologically classified according to their properties in starting zone, track, and runout zone, most of these classifications are not significant for the discussion of internal processes.

The significant classifications address the following properties:

- Water content in snow:
  One differentiates between dry avalanches and wet avalanches. The differentiation is somewhat imprecise because no clear rules can be formulated.

- Form of motion:
  This classification is highly dependent on the water content (which influences internal cohesion) and terrain properties.
    - Flow avalanches: These avalanches flow on the ground or weak-layer. They entrain less air, which means that no air-snow-suspension cloud is formed.
They consist of either wet, moist, or dry snow. Their velocities are medium (dry) to low (wet).

- **Powder avalanches**: They consist only of very dry snow and thus produce a large air-snow-suspension cloud which flies above the solid snow flow on the ground. This avalanche-type reaches the highest speeds, and develops from flow avalanches. The minimum necessary track gradient is 30°.
- **Mixed avalanches**: Most avalanches feature characteristics of both flow and powder type. Again, the classification is rather vague and requires experience.

### 3.3 Internal processes [HTL, p. 66ff.]

Still, very little is known about the complex interactions of physical processes within an avalanche. This is mainly due to the great variability of snow. Furthermore, it is very challenging to analyze and collect data of avalanches in real life.

All internal physical processes have a decelerating effect. They constitute the frictional force \( \vec{F}_r \) and are a result of internal shear, deformation, and momentum exchange. As the blocks of snow collide, deform, reconsolidate, melt or fluidize in air, the kinetic energy emerging from gravity is diffused.

WestWide Avalanche Network [L12] lists the following five forces:

- \( \vec{F}_{r1} \): Sliding friction between the avalanche and the underlying snow or ground
- \( \vec{F}_{r2} \): Internal dynamic shear resistance due to collisions and momentum exchange between particles and blocks of snow
- \( \vec{F}_{r3} \): Turbulent friction within the snow/air suspension
- \( \vec{F}_{r4} \): Shear between the avalanche and the surrounding air. This friction is responsible for formation of air-snow-suspensions
- \( \vec{F}_{r5} \): Fluid-dynamic drag at the front of the avalanche

Although all these decelerating forces are present in an avalanche, their relative importance depends on the avalanche type and location within the flow.

Flow avalanches consist of strongly bonding snow, which does not easily disintegrate into a small-particle flow, but contains rather bigger blocks of snow (10 – 100cm). These
blocks slide and roll down the slope and never really develop into a suspended powder cloud or a smooth flow. Therefore, \( \vec{F}_{r1} \)'s sliding friction has a high influence. This is because sliding friction rather affects solids than fluids. Furthermore, since the snow is strongly coherent, it does not entrain much air. Thus, the distance between the individual particles rests small, and more collisions and momentum exchange occur. Therefore, \( \vec{F}_{r2} \) is also of great importance. \( \vec{F}_{r3}, \vec{F}_{r4}, \vec{F}_{r5} \) are less important, because their effect is highest on air-snow-suspensions.

On the other hand, powder avalanches are mainly subject to the forces \( \vec{F}_{r3}, \vec{F}_{r4}, \vec{F}_{r5} \). The dryness of the snow makes it less coherent [L13]. Therefore, more air is entrained, particle distance increases, and the avalanche gains the properties of a "true" flow. As a result, \( \vec{F}_{r1} \) and \( \vec{F}_{r2} \), which are the stronger friction forces, diminish in importance. Consequently, powder avalanches reach higher velocities than flow avalanches. Because the turbulent drag of \( \vec{F}_{r5} \) is proportional to the square of the velocity [CL, p. 65] of the avalanche, its effect increases as the avalanche accelerates.

In the end, for all types of avalanches, each friction force plays its part in slowing down an avalanche to a full stop. The assumptions made above are still highly theoretical and require experimental proof.

Because the forces acting on avalanches, as well as their interactions, are so complex and varied, it has yet been impossible to create a complete, precise, and reliable model of avalanche movement. This, however, remains to be the ultimate goal of avalanche dynamics research.

4. Avalanche modeling

4.1 Nature and purpose of models

First of all, I want to clarify the term "model".

The explanation provided by the SLF [L14] seemed so plausible to me, that I decided to quote it directly:

"One of the goals of scientific research is to explain past observations, control natural processes or predict future outcomes. Avalanches are [...] very complex and involve many different and interconnected components, [which] makes it very difficult to understand
4. Avalanche modeling

such a phenomenon in its entirety. Usually, not all aspects of such a phenomenon are equally important for a particular question. It is therefore often possible and beneficial to simplify and abstract the reality in order to make it more convenient to handle. This is what is called a model."

The more abstraction and simplification is applied to a phenomenon, the lower the complexity of the resulting model. And the complexity of a model determines its computability. Some models allow for calculations by hand, whereas other demand for the immense capacities of supercomputers. Unfortunately, lower complexities result in less accuracy. The desired level of accuracy depends on the question asked.

What is the question in the case of avalanche dynamics? What outcomes do we expect from an avalanche model?

These desired outcomes are called characteristic criteria of avalanche movement [HTL, p. 63]. They are all of practical use for risk analysis such as hazard zone mapping. They include:

- Flow height \( h \) (m)
- (maximum) velocity \( v_{\text{max}} \) (m/s)
- Density \( \rho \) (kg/m\(^3\))
- Impact pressure \( p \) (Pa)
- maximum runout-distance \( s \) (meter)

4.2 Types of models [L14 and HTL, p. 75f.]

There are two types of models that can be used to obtain the desired characteristic criteria mentioned above:

1. Physical-dynamic models: Physical laws govern the different constituents of the model.
2. Statistical-topographic models: Statistical relationships between the different constituents are established using data from past avalanche events.

In this paper, I focus on the physical-dynamic models. In general, they provide more accurate results than statistical approaches. However, they also require much more calculation.
4.3 Physical-dynamical models [HTL, p. 66ff.; L15]

In order to establish a physical model, the avalanche movement is simplified to its fundamental governing physical laws. Avalanches can formally be regarded as a moving continuous medium. “A medium is continuous, if it completely fills the space that it occupies, leaving no pores or empty spaces, and furthermore if its properties are describable by continuous functions” [L. E. Malvern, 1969].

Obviously, this definition ignores the fact that all materials are made of atoms, which contain empty spaces. The medium snow clearly does not satisfy this definition. Hence, we note that a first abstraction from reality has been made.

Regarding a material as a continuum enables accurate modeling possibilities [L16]. Fundamental physical laws like the conservation of mass, energy, and momentum can now be applied to the avalanche movement. All these laws can be expressed by partial differential equations, which describe the behavior of the medium. In order to obtain the desired characteristic criteria, provided adequate initial conditions, from this system of equations, it first has to be "closed". This is done by adding information about the particular material's behavior in the form of constitutive relations, also called rheology. It describes the relation between the deformation tensor D and the stress tensor σ. In other words, it describes the deformation of a material in relation to an acting stress.

These relations are expressed by constitutive equations. In fluid mechanics, they describe the material’s behavior in more or less simplified form, depending on their mathematical complexity. The simplest constitutive equation is the one of the linear relationship between σ and D of so-called Newtonian Fluids. Avalanches technically consist of the most common Newtonian Fluid: water. However, as the water freezes to form porous snow, the Newtonian properties disappear and its rheological behavior becomes more complex. Furthermore, each type of snow requires its own constitutive equation, which can only be found experimentally. Unfortunately, this is not yet technically feasible.

Consequently, the constitutive behavior of flowing snow is grossly simplified in all physical avalanche models (such as in section 4.3.2). Improving the constitutive equations is one of the key focuses of avalanche dynamics research.
4.4 The Voellmy-Salm model [L15; HTL, p. 72ff.; L17; HTL, p. 76ff.]

The Voellmy-Salm model is a well-known and regularly used example of a physical-dynamical avalanche model.

In the older models before the Voellmy-Salm model, avalanches were regarded as one single sliding block of snow, decelerated only by dry Coulomb friction. They enabled somewhat useful assessments of avalanche danger areas.

As the demand for precision and reliability grew, new physical concepts were added to existing models. Fluid and continuum mechanics, like described above, were added in order to account for the inner processes of avalanches.

The Voellmy-model (predecessor of the Voellmy-Salm model) was created by Dr. Adolf Voellmy, a Swiss building engineer, in 1955. Dr. Voellmy analyzed and reconstructed the damages and impact pressures of the avalanche which had destroyed the village Blons in Austria. In his model, he considered an avalanche to be a hybrid of continuum and granular flow. He then implemented a combination of the dry friction of Coulomb and the Chezy-formula. The Chezy-formula originates from hydraulics and describes the velocity of a uniform, open channel flow [PHaWRE, p. 170]. This shows a typical scientific approach for acquiring new knowledge: trying to understand new phenomena using knowledge from other, related phenomena. His approach resulted in the constitutive relation known as the 2-parametric-voellmy-fluid-friction model (or Voellmy-friction model), which still serves as the basis for most modern, numerical avalanche models.

However, the Voellmy-Model was not able to calculate run-out distances and flow velocities. Dr. Bruno Salm then solved this problem in 1970 by extending the model. He differentiated between release-, acceleration-, and runout-zone. Each zone is characterized by its mean gradient. The resulting model is named Voellmy-Salm model.

In 1990, the SLF published a practice-orientated manual based on the Voellmy-Salm model. It is used worldwide with great success to calculate runout distances and to scale protective measures.

The Voellmy-Salm model is capable of calculating velocity, height, runout distance and impact pressure of flow avalanches (but not powder avalanches). It relies on the following assumptions:
4. Avalanche modeling

- Constant flow rate along the track
  This is a significant simplification, since we know that avalanches entrain considerable amounts of snow during their decent.
- Snow is deposited only in the runout zone. This assumption is rather consistent with reality.
- Steady flow in the track. In reality, avalanches continuously accelerate and decelerate in relation to changes in topography.
- Microstructure and forces within the avalanche are ignored. Flow is considered incompressible. Consequently, velocities and densities are regarded uniform for the whole flow. In reality though, flow velocities differ significantly between front, body, and tail of the flow.
- Flow heights vary only slightly along the track.

Combining these assumptions, this model treats the avalanche as a body of uniform velocity and density, descending in the track with constant flow rate (thus without entraining or depositing snow) until it reaches the runout zone. This is known as a deterministic-dynamic block model. In the runout zone it decelerates, deposits snow, and reaches a standstill.

Because the model is based on the Voellmy-friction model (or Voellmy-rheology), the frictional resistance is divided into two parts:

1. A dry-Coulomb type friction: \( \mu \) (unitless) \([L20]\)

   This friction accounts for the solid phase of an avalanche. An avalanche is in a solid phase when its granular properties \([C. Ancey et al., 2004]\) (blocks of snow rolling and colliding) exceeds its viscous, fluid properties (when an avalanche has developed into a smooth flow). Typically for dry frictions, \( \mu \) is (mathematically) independent of velocity and scaled with normal force \( F_N \). However, since it pertains to snow fluidity \([C. Ancey et al., 2003]\), it is indeed dependent on flow velocity. Snow develops into a flow at higher speed and regains solid properties as is decelerates. It thus theoretically varies during avalanche descent. Its influence is highest at low flow speeds. It needs to be calibrated to a constant value, according to the expected avalanche properties for the examined path. The SLF suggests the following values \([L21]\):
4. Avalanche modeling

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>Condition</th>
</tr>
</thead>
</table>
| 0.155     | - extreme avalanches (rare avalanches with very large volumes > \( 10^6 \text{ m}^3 \))
|           | - higher altitudes, dry cold snow                                          |
|           | - flow depth > 1 to 2m                                                    |
| 0.20      | - as above but for dry snow at higher temperatures, lower altitudes       |
| 0.25 - 0.30 | - smaller avalanches with lower mean return periods and volumes < \( 10^4 \text{ m}^3 \) |
| 0.30      | - wet snow avalanches of any size                                          |

Table 1: Proposed values for \( \mu \)

From this list, together with the explanations above, one can deduce the following relationships:

- The heavier (= the more water content) in relation to size, the bigger is \( \mu \). This is because \( \mu \) is scaled with \( F_N \).
- The bigger the size, the smaller is \( \mu \) (because bigger avalanches in general gain higher speeds and hence more fluid properties).
- The more water content, the higher the cohesion, the bigger is \( \mu \). With more cohesion, the avalanche resembles more of a solid, sliding block than a turbulent fluid flow.

2. A viscous-turbulent friction: \( \xi [\text{m/s}^2] \)

It accounts for the resistance during the fluid phase (flow at high velocities). Voellmy introduced it as a turbulent drag force using hydrodynamic arguments. Turbulent drag, which is proportional to squared velocity [CL, p. 65], is generally produced by obstacles. \( \xi \) is thus dependent on the path’s geometry (roughness and canalization). Here are the values proposed by the SLF:
4. Avalanche modeling

<table>
<thead>
<tr>
<th>$\xi$ [m/s$^2$]</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>- low roughness&lt;br&gt;- moderately canalized (ratio width: flow height &gt; 10 : 1)</td>
</tr>
<tr>
<td>500-600</td>
<td>- big roughness&lt;br&gt;- strongly canalized</td>
</tr>
<tr>
<td>400</td>
<td>- flowing through forest</td>
</tr>
</tbody>
</table>

Table 2: Values proposed for $\xi$

These recommendations are rather sparse, choosing the proper values for $\mu$ and $\xi$ requires a lot of experience.

The Voellmy-Salm model is an empirical model, because these friction parameters can only be estimated empirically. Since the snow’s rheological behavior is still largely unknown, such friction parameters cannot be physically determined. In the case of the proposed values above, they have been calibrated to measured runout distances of extreme avalanches, as well as observed frontal flow velocities in small- and large-scale experiments. This model is based on mass and momentum balance equations.

The mass balance equation states that the flow rate $Q$ stays constant until runout zone. It is expressed as

$$Q = Q_0 = B_0 \cdot d_0 \cdot v_0$$

(1)

where $d_0$ [m] initial fracture height (flow height in release zone), $v_0$ [m/s$^2$] representative velocity when leaving the release zone, $B_0$ [m] avalanche width in release zone. The process of deriving $d_0$ is not further explained. It involves a correlation to snowfall during the last three days. $B_0$ is obtained from observations in field or on the map. In order to find $v_0$, the momentum equation is solved.

The momentum equation is expressed as

$$m \frac{dv}{dt} = F_a - F_r$$

Implementing the Voellmy-friction model, this becomes
4. Avalanche modeling

\[ m \frac{dv}{dt} = mg \sin(\varphi) - \mu mg \cos(\varphi) - \frac{mgv^2}{\xi d} \]  

where \( m \) stands for avalanche mass [kg], \( \varphi \) for slope gradient [°], and \( g \) for the gravitational constant. The middle term represents dry friction with coefficient \( \mu \) and \( F_N = mg \cos(\varphi) \). The right term stands for turbulent friction with coefficient \( \xi \) [m/s²] and proportional to velocity squared. The flow’s friction increases as \( \mu \) increases and \( \xi \) decreases.

Voellmy-Salm’s model is rooted in the hydraulic theory. Therefore, "the avalanche is modelled as a fluid which accelerates very quickly from rest to a steady, terminal velocity" [R. Perla et al., 1980]. The velocities calculated with this model are thus regarded constant, with \( \frac{dv}{dt} = 0 \). Equation 2 becomes

\[ \frac{v^2}{\xi d} = \sin(\varphi) - \mu \cos(\varphi) \]

by canceling out \( mg \). The velocity \( v_0 \) can be calculated with this equation:

\[ v_0 = \sqrt{\xi d_0 \left( \sin(\varphi_0) - \mu \cos(\varphi_0) \right)} \]

with \( \varphi_0 \) standing for mean gradient in the release zone.

At this point we observe that the results of this model strongly depend on parameters \( d_0, \mu, \) and \( \xi \).

Now, point P is determined. It is the point with critical gradient \( \varphi_k = \arctan(\mu) \). It marks the beginning of the runout zone with gradient \( \varphi_s < \arctan(\mu) \), where the avalanche decelerates. The velocity at point P (or any other point in the path) is calculated by

\[ v_p = \sqrt{\xi d_p \left( \sin(\varphi_p) - \mu \cos(\varphi_p) \right)} \]

where \( \varphi_p \) stands for the gradient above P (measured for a stretch of 100-200m).

Because \( d_p \) cannot be known for this point, we make use of \( Q \). Since \( Q \) is always constant,

\[ Q = Q_0 = Q_p = B_p \cdot d_p \cdot v_p \]

\[ d_p = \frac{Q}{B_p \cdot v_p} \]
where $Q$ is known and $B_p$ can be derived from topography.

To finally calculate runout distance $s$, this model provides the following equation:

$$s = \frac{d_s \xi}{2g} \ln \left(1 + \frac{v_p^2}{V^2}\right)$$

where

$$d_s = d_p + \frac{v_p^2}{10g}$$

and

$$V^2 = d_s \xi (\mu \cos(\phi_s) - \sin(\phi_s))$$

$V$ (auxiliary variable) stands for the mean avalanche velocity and $\phi_s$ for the mean gradient in the runout zone.

Runout distance $s$ is measured from point P to the furthest reaching point of the avalanche. This model has numerous advantages. It is well validated and convenient to calculate "simple" avalanches taking very little time. It therefore fits the purpose required by the research question.

However, it is criticized in the scientific community, especially for the *a priori* selection of point P based on a purely empirically chosen value for $\mu$ [R. Perla et al., 1980]. In general, the dependence of this model on unsatisfactorily known parameters ($d_0, \mu,$ and $\xi$) leads to many insecurities and *ad hoc* assumptions [L22].
5. Practical application of avalanche dynamics

5.1 Motivation and goal

Up to this point, my work has been purely theoretical. I provided an overview on the phenomenon of snow avalanches, as well as an in-depth discussion of the physics of avalanche dynamics.

Additionally, I also wanted my work to include a practical application of this new knowledge. I therefore decided to conduct an experiment which investigates the physics of avalanche dynamics from a different perspective.

While doing research on avalanche dynamics, I came across the SLF's snow chute [L18]. They use this 30-meter-long and 2.5-meter-wide chute to analyze the flow behavior of artificial avalanches by means of highspeed cameras.

Inspired by this, I decided to construct my own, smaller chute. My work yet had to be written and submitted before the winter season, which means that the material studied could not be snow.

However, snow avalanches are only one type of avalanches. Avalanches are gravitational mass movements and can thus consist of many other materials, such as ice, debris, rock, and mud. Physically, all these natural hazards are closely related [L19]. They all have granular and fluid properties and some of them can be modeled using continuum mechanics. It is thus reasonable to apply physical concepts of snow avalanches to other types of mass movements.

Therefore, I decided to use the material "sand" for my experiment. This initial situation opened up an interesting research question: "Can snow avalanche models also be applied to sand avalanches"?

I will investigate this question by making use of the most important characteristic criteria that all mass movements have in common: maximum runout distance. Hence, I needed a snow avalanche model that is capable of calculating this criterion. I completed a three-day-internship at Etufor SA in St. Léonard, Switzerland. There, Mr. Thierry Darbellay, graduate forestry engineer (M.Sc. ETHZ), taught me how to use the
Voellmy-Salm model, based on the guidelines published by the SLF in 1990 [B. Salm et al., 1990]

This is how I arrived at the research question stated at the beginning of the paper.

5.2 Experiment

I will investigate the effect of different inclination gradients (denoted $\varphi$) (see Picture 6) of the acceleration zone on the runout distance (denoted $s$) of sand flows. I will then try to find a configuration of the parameters $\mu$ and $\xi$, so that the calculations of the Voellmy-Salm model match the measured data. If such a configuration can be found, this would support the research question affirmatively.

5.2.1 Chute

The chute was designed for simple calculations with the Voellmy-Salm model.

I divided it into two zones (see Picture 2). The acceleration zone (length: 215 cm, width:100 cm) corresponds to the model's release zone and path. Its gradient can be altered from 0 to 90°. It is connected to the runout zone via two hinges (length: 150 cm, width: 100 cm), with a gradient fixed at 5°. There is a compartment for sand (length: 35 cm, width: 100 cm) above the acceleration zone. Its height is restricted to 5 cm by a removable and fastened wooden lid. The released sand's volume is hence $1.75 \cdot 10^{-3} \text{ m}^3$.

The upper part, containing the sand compartment and acceleration zone, is 250 cm long. The wooden plate separating the sand compartment from the acceleration zone is also removable.

Both zones are confined on each side by a 20 cm high sidewall. The exact same material is used for both zones and sidewalls. The lower plate is placed on a crossbeam, which again is placed on a stone plate with height 2.5 cm, so that the angle of 5° was established.
Picture 1: The chute

Picture 2: Zones of the chute

Picture 3: Sand compartment
5. Practical application of avalanche dynamics

The upper plate can be pulled up by a cord (see Picture 3). It is lead through a carabiner which is connected to the crossbeam of a wooden swing, which was repurposed for the experiment.

The upper end of the upper plate features two carabiners, which are connected to two chains hanging from the swing’s crossbeam. The respective chain-link was marked so that the desired angle could be established.

A GoPro camera was in the middle of and perpendicular to the acceleration zone. It was held in place by a construction of two wooden poles.

5.2.2 Idea behind the design

The chute should allow the sand to be released in a manner similar to a slab avalanche. It is important that the sand has a uniform height along the whole width and length. It then resembles a snowpack. The fastened lid ensures that the sand does not slide forwards against the separating plate as the angle is increased, because this would change $d_0$. $d_0$ is kept constant at 5 cm for the entire sand body.

The point P is found at the beginning of the runout zone, with an inclination below the critical angle. Runout distance is measured from this point on.

When the separating plate is pulled up quickly, the entire sand simultaneously starts sliding downwards, just like a slab avalanche after the weak layer has broken.

The width is kept constant along the whole path to simplify calculations.

I decided to minimally incline the runout zone (5°). I wanted to avoid that the sand is stopped abruptly and would pile up due to a large gradient-difference. When this happens, physical processes take place which are not accounted for in the Voellmy-Salm model. In nature, such transitions to smaller gradients are a lot smoother. However, I could not incline the runout zone too much, because the sand had to stop within the 150 cm.

5.2.3 Methodology

Firstly, the upper plate is lowered to the ground. The sand (25,3 kg) is filled into the sand compartment. The lid is placed on the sand compartment and held in place by four sliders.

Then, the upper plate is pulled up by one person. An assistant connects the two carabiners of the upper plate to the two chains at the indicated chain-link for the desired angle. At
this point, the angle formed between the upper and lower plate is measured using a self-built, lockable angel gauge (see Picture 4). If necessary, the lower plate and the crossbeam underneath are moved forwards or backwards until the desired angle is exactly met.

In only one run per angle, the sliding sand is recorded using the GoPro® camera. Then, prior to releasing the sand, 3 easily discernible, colored objects are dug into the sand in the compartment. They are needed for subsequent velocity analysis (see section 5.3.1).

Next, two persons simultaneously lift the plate, which separates the sand compartment from the acceleration zone, about 15 cm, so that the whole sand height is released instantaneously. After the sand has come to a full stop, the runout distance is measured and recorded (see Table 3). Next, the sand in the chute is collected and placed in two buckets. Remaining sand is wiped off the chute using a broom. The buckets are weighed and sand is refilled to 25.3 kg. One full run is now completed, and the experiment is repeated.

The data for $\phi = 35^\circ$ and $40^\circ$ was collected on one day and for $\phi = 45^\circ$ and $50^\circ$ on another.

5.2.3 Variables

➢ Independent variable

Angle between upper plate and horizontal line (see Picture 6).

Denoted $\phi = \{35, 40, 45, 50\}^\circ$. The angles are verified using the self-built, lockable gauge. The gauge is first placed on a A3 paper on which angle indications were drawn (see Picture 5), using a large triangle ruler. The gauge is adjusted so that it fits the angle $A = (180^\circ - \phi) + 5^\circ$. The gauge is locked and placed between the lower plate and upper plate (see Picture 4). The whole chute is moved until the gauge parallelly touches both parts of the chute. $5^\circ$ are added to $A$ because the lower plate is inclined by $5^\circ$. 
5. Practical application of avalanche dynamics

➢ Dependent variable

Runout distance, denoted $s \text{ [cm]}$, measurement uncertainty $= \pm 1 \text{ cm}$

It means the distance between the beginning of the runout zone and a point which meets these criteria (see Picture 7):

1. It is part of the sand flow
2. It is the farthest away from point P.
3. On a straight line between this point and the tail of the sand body, the wooden plate is not visible.

Picture 7: measuring runout distance

➢ Controlled variables

- Amount of sand = 25.3 kg, measurement uncertainty = ± 0.1 kg.
  
  In order to determine this weight, sand was initially filled into the compartment so that it exactly filled up to a height of 5cm. The sand was taken out again and weighed.

- Angle of lower plate = 5°.
  
  This angle was established using a crossbeam placed on a wooden plate. The height of the crossbeam was calculated using trigonometry. It ensures an equal angle for each run. The angle was initially verified using the above-mentioned gauge.

- Air humidity in garage ≈ 50%, measured with hygrometer, uncertainty = ± 1%.
  
  The sand was stored there between the two data collection days. This assures fairly similar sand humidity.
5.3 Analysis

5.3.1 Velocity analysis

The Voellmy-Salm model regards the velocities as constant at each point along the acceleration zone. Since the calculated runout distance strongly depends on the calculated velocity at point P, the sand flows should attain a constant velocity before they reach point P. If not, a comparison between observed and calculated values is not entirely meaningful.

I will thus explain how I measured the velocities of the sand flows. A GoPro® camera was installed in the very middle of- and perpendicular to the acceleration zone. For each angle $\phi$, one run was recorded. The video was then analyzed using the mobile application Vernier Video Physics [L23]. This allows you to track the location of a specific point on a moving object and provides information on velocity and position at every given time. Since the sand flows elongated quite quickly after release, it was impossible to track a specific point. I thus buried easily visible PET bottle lids (in different colors) into the sand, in the sand compartment. It was placed at the same position each time, which ensured that it showed up at the approximately equal position in each flow.

Picture 8: Yellow lid's position along the acceleration zone (red points).  
Picture 9: coordinate system and scaling
Picture 8 depicts how the yellow lid was located in the application. It was also necessary to define the point of origin and the real length of a specific feature (here acceleration zone length). This is shown in Picture 9. The application then produced the following diagrams:

The upper diagram in Pictures 10, 11, 12, and 13 show the position of the lid along the acceleration zone (y-axis) at each time (x-axis). Each of these sequences are curved at the beginning and become rather linear towards the end. A linear change in displacement with respect to time implies a constant velocity. We note that the curvature is stronger for larger angles. Hence, the sand flow reaches a constant velocity faster when the inclination is less steep. This makes sense, since the downward forces decrease with a decreasing angle. As a result, the friction forces equate the downward forces faster, and when $F_{net} = 0$, $a = 0$ a constant velocity is attained.

In picture 12, the points between the time 11.3 s and 11.5 are missing because the lids were temporarily covered by sand.
5. Practical application of avalanche dynamics

The lower diagrams display the instantaneous velocity. We note that the velocity increases rather linearly in the first part. Then, the change in velocity -with respect to time- decreases and the sequence flattens. In each lower diagram, the sequences tend to become flat towards the end. This indicates a rather constant velocity. On the other hand, the last 3-5 points rise rapidly, showing an abrupt deceleration. This deceleration occurs when the main, middle part of the flow reaches the runout zone, where the smaller inclination leads to a decreased downwards force. Additionally, some kinematic energy is lost when the sand is compressed during the transition between the two inclinations.

However, by this method, only the velocities in the middle part of the flow were measured. Since the flow was elongated, it is certain that the velocities of the frontal part were considerably higher and probably never constant. This is critical, because it is especially this part which eventually marks the maximum runout distance. Nevertheless, for the purpose of this experiment, it is sufficient to know that the major quantity of sand nearly reached a constant velocity.

As a conclusion, none of the flows reach an absolute constant velocity. However, they exhibit a clear tendency towards constant velocity, because the change in velocity became very small towards the end. The observed runout distances are thus regarded significant.

5.3.2 Runout distance analysis

<table>
<thead>
<tr>
<th>Angle $\phi$ [°]</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>Runout distance $s$ [cm] ± 0.5 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>75</td>
<td>92</td>
<td>110</td>
<td>129</td>
</tr>
<tr>
<td>2</td>
<td>74</td>
<td>90</td>
<td>108</td>
<td>127</td>
</tr>
<tr>
<td>3</td>
<td>77</td>
<td>89</td>
<td>114</td>
<td>124</td>
</tr>
<tr>
<td>4</td>
<td>76</td>
<td>92</td>
<td>112</td>
<td>129</td>
</tr>
<tr>
<td>5</td>
<td>74</td>
<td>91</td>
<td>114</td>
<td>126</td>
</tr>
<tr>
<td>Mean value</td>
<td>75,2</td>
<td>90,8</td>
<td>111,6</td>
<td>127,0</td>
</tr>
<tr>
<td>Standard sample deviation</td>
<td>1,30</td>
<td>1,30</td>
<td>2,61</td>
<td>2,12</td>
</tr>
<tr>
<td>Standard error of the mean</td>
<td>0,58</td>
<td>0,58</td>
<td>1,17</td>
<td>0,95</td>
</tr>
</tbody>
</table>

Table 3: Presentation of quantitative data
Table 3 shows the total amount of quantitative data collected. Five runs were conducted for each angle $\varphi$, and recorded in an Excel spreadsheet. Because $s$ was measured to the nearest centimeter, the error = $\pm$ 0.5cm. The standard deviation and standard error of the mean for each angle is fairly small. This indicates that each run was conducted in a rather consistent and equal manner, and assures the significance and reliability of the derived means.

Graph 1: Mean runout distance $s$

Graph 1 displays the mean runout distance for each angle. It shows that the runout distance increases as the acceleration zone's angle increases. This is reasonable, because as the inclination increases, the downwards force increases. As a consequence, the sand accelerates at a higher rate, gains more speed and requires a longer distance to come to a full stop. The difference between $\varphi = 35^\circ$ and $40^\circ$ and $\varphi = 45^\circ$ and $50^\circ$ is nearly equal (15.6cm and 15.4cm). However, the difference between $\varphi = 40^\circ$ and $45^\circ$ is equal to 20.6cm and thus does not fit the trend. This inconsistency might be explained by the fact that the data for $\varphi = 45^\circ$ and $50^\circ$ was collected on another day. Different weather conditions (slightly drier air) might have caused the sand to flow faster, since the plate's surface was less moist. Error bars are included in Graph 1, but are too small to be visible.

I will now compare the runout distances -which I have calculated with the Voellmy-Salm model- with the corresponding experimental results. My goal is to find out if the
experimental runout distances could also have been accurately predicted by using this model.

First, the experimental data needs to be statistically processed, in order to account for the standard error of the mean runout distances. The statistical method of “Fitting a straight line” (linear regression) to the means of the measured $s$ at the four values of $\phi$ is used. The procedure is explained in [DA, chapter 9.3] and conducted in the Appendix.

This procedure produces the following improved values:

<table>
<thead>
<tr>
<th>Angle $\phi$ [°]</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runout distance $s$ [cm]</td>
<td>74.6</td>
<td>92.0</td>
<td>109.4</td>
<td>126.9</td>
</tr>
</tbody>
</table>

Table 4: Improved mean runout distances

We note that the improved means in Table 4 differ only slightly from the original means in Table 3, yet the standard error of the means have been significantly reduced for all values of $\phi$. Therefore, the new means are statistically more reliable and will thus be compared to the calculated runout distances in Graph 2.

I implemented the calculations into an Excel spreadsheet. The parameters in Table 5 had to be set for the calculations. They correspond to the dimensions of the chute and the independent variable $\phi$. All angles had to be converted into radian form.

<table>
<thead>
<tr>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
</tr>
<tr>
<td>$B_0$</td>
</tr>
<tr>
<td>$B_P$</td>
</tr>
<tr>
<td>$\phi_0$</td>
</tr>
<tr>
<td>$\phi_P$</td>
</tr>
<tr>
<td>$\phi_S$</td>
</tr>
<tr>
<td>$\xi$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
</tbody>
</table>

Table 5: Parameters used for runout distance calculation
Unfortunately, the values for the friction parameters $\mu$ and $\xi$ are not known. I cannot use the values proposed in section 4.3.2, since they were calibrated for large-scale snow avalanches. Although chute experiments with granular materials have been conducted before, I was unable to find values for friction coefficients that are applicable to my experiment. This is due to the fact that these coefficients depend on many factors such as type of material, type of sand, sand humidity and especially sand mass. Unless the exact same sand, mass and chute surface material were used for previous experiments, the findings will only provide an approximate idea of the magnitude of the friction coefficient values.

I thus had to conduct a back-analysis of my experimental results. Such calibration exercises provide estimates of input parameters by inverse analysis [D. Mancarella, 2010]. In other words, I tried to find a configuration of $\mu$ and $\xi$ values, so that the output fits the observed runout distances for each angle $\phi$. If such a configuration could be found, I would be capable of accurately predicting the runout distance of all future sand flows on my specific chute. This back-analysis was also applied to case studies of larger snow avalanches. The values of $\mu$ and $\xi$ proposed in section 4.3.2 are the result.

In Excel, I created a line chart that displayed both the improved observed and calculated runout distances for each $\phi$. Each of the two data sets thus has its own line. By hand, I altered the values for $\mu$ and $\xi$ until the two lines coincided as close as possible.

![Graph 2: Line chart of improved mean observed and calculated runout distances $s$](image-url)
The best conformity was achieved with $\xi = 255 \text{ m/s}^2$ and $\mu = 0.29$. This configuration resulted in the calculated runout distances shown in Table 6. It is notable that both sets seem to increase linearly. This is surprising, given that the function for calculating $s = f(\phi)$ contains strongly nonlinear cosine and logarithmic functions. The calculated linear behavior of $s$ in the range from $\phi = 35^\circ$ to $\phi = 50^\circ$ supports our previous decision of fitting a linear regression line to our observed data by the method of least squares [DA, chapter 9.3]. From this graph, one might confidently conclude a general linear relationship between runout distance and inclination. Nevertheless, this assumption is not statistically backed, because $\xi$ and $\mu$ were found by eye using try-and-error. Thus, these values served only as a first approximation and required statistical validation. To do that, I made use of "The general case of least-square fitting" [DA, chapter 9.10]. This method requires that the function $s = f(\phi)$ behaves fairly linearly in the region around the first approximation. Equation 9.10.17 of this method then produced the new values of $\xi$ and $\mu$: 255,110 m/s$^2$ and 0,29425. Because this method produced values very similar to our first approximation, it had now been statistically validated that $s = f(\phi)$ behaves linearly or fairly linearly with and around input parameters $\xi = 255 \text{ m/s}^2$ and $\mu = 0.29$. As a result, the values $\xi = 255 \text{ m/s}^2$ and $\mu = 0.29$ and consequently the calculated runout distances in Table 6 are regarded as statistically significant.

<table>
<thead>
<tr>
<th>Angle $\phi [^\circ]$</th>
<th>Calculated</th>
<th>Mean observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>77.4</td>
<td>74.6 ± 0.50</td>
</tr>
<tr>
<td>40</td>
<td>94.0</td>
<td>92.0 ± 0.36</td>
</tr>
<tr>
<td>45</td>
<td>109.7</td>
<td>109.4 ± 0.50</td>
</tr>
<tr>
<td>50</td>
<td>124.4</td>
<td>126.9 ± 0.79</td>
</tr>
</tbody>
</table>

Table 6: Table of calculated and mean observed runout distances

In Table 6 it becomes apparent how well the calculated $s$ fit the experimental data when using the input parameter $\xi = 255 \text{ m/s}^2$ and $\mu = 0.29$. The percentage errors [Weissstein, Eric W.],

$$PE = \frac{s_c - s_o}{s_o} \cdot 100\%$$
where $s_c$ stands for calculated $s$ and $s_o$ for observed $s$ (considered "true"), range from minimum 0.274% to maximum 3.75%. Such small errors strongly support my hypothesis that the runout distances of small-scale sand avalanches can be accurately predicted using the Voellmy-Salm model and appropriate values for the friction coefficients $\xi$ and $\mu$.

5.4 Discussion

First of all, I would stress that the obtained values cannot be applied to sand flows in general. The values achieved are only relevant for this specific experiment. Nevertheless, they are still scientifically helpful. With my chute, it would now be possible to conduct more complex experiments involving e.g. protective measures. My friction values could then be used for the calculations and the process of back-analysis would be obsolete. Additionally, they provide a limited insight into the rheological behavior of sand.

Second of all, the experiment features a considerable limitation. In section 4.3.2 it is explained how the turbulent friction $\xi$ is most effective during the fluid-like state of the flow. For snow avalanches, this state only occurs at high velocities, typically in the avalanche path. Consequently, the $\xi$ coefficient mainly influences the flow's velocity. On the other hand, the $\mu$ coefficient takes over when the flow assumes a solid-granular phase, which occurs at low velocities. Hence, $\mu$ dominates when the flow is close to coming to a stop, which is the case in the runout zone [H. Y. Hussin et al., 2012]. A calibration of the $\xi$ coefficient based solely on runout distance is thus not very reliable (LRM, p. 117, 123). Although the derived configuration of $\mu$ and $\xi$ produces the desired outcomes, the principle of equifinality must be considered. It states that "there are many ways or paths [in our case parameter configurations] that can lead to the same end [in our case runout distance]" [von Bertalanffy, 1968]. It is thus crucial to compare the results of calibrations to many other control parameters from observations, like spot velocities and flow height, flow duration and deposition thickness [LRM, p. 123]. The more parameters are taken into account, the fewer input parameter configurations exist. In most studies, at least the runout distance and velocities are compared [D. Rickenmann, 2005]. Unfortunately, my simple measurement method was not precise enough to additionally compare observed and calculated flow velocities.

Nevertheless, I will interpret both input values and compare them to other scientific background.
5. Practical application of avalanche dynamics

Previously, I have already deduced two general relationships for $\mu$: the smaller the avalanche, the bigger $\mu$ and the bigger the relative weight, the smaller $\mu$. These rules would fit the high value of 0.29 which I have obtained for $\mu$. Our sand flow is extremely small, but its relative weight is high. It rather resembles a wet snow avalanche ($\mu = 0.3$) than a powder avalanche ($\mu = 0.155$). Yet, such comparisons are quite trivial, since we compare completely different materials and dimensions.

Further, the very low value of 255 m/s$^2$ for $\xi$ does not make any sense when being compared to snow avalanches. $\xi = 400$ m/s$^2$ is assumed when the avalanche flows through a forest, and clearly my chute does not exert such high friction. However, low velocities correspond to small $\xi$ values. For $\phi = 50^\circ$, the maximum velocity was approximately 3.7 m/s, whereas a wet snow avalanche easily reaches 10-20 m/s [HTL, p. 65].

However, parameter calibration from studies regarding debris flows are more meaningful. In general, the Voellmy rheology produces good results for debris flow back-analysis. It is one of the most widely used rheology in debris flow modeling [H. Y. Hussin et al., 2012]. Quan Luna et al. (2010) compiled a data base of 152 debris flows which were back-analyzed with the Voellmy rheology and established probability density functions for both friction coefficients. For $\mu$, the highest frequency was found between 0.05 and 0.2. For $\xi$, the values were most frequent between 150 and 600 ms$^2$. These ranges might appear quite large, yet it should be considered that literature discusses values ranging from 0.01 to 0.7 for $\mu$ and from 100 to 3000 ms$^2$ for $\xi$ [H. Y. Hussin et al., 2012].

My $\mu$ of 0.29 is not included in the mentioned range. However, since the flow is a lot smaller than the debris flows of the database, the slightly larger value seems reasonable. My $\xi$ of 255 ms$^2$ conforms to the lower part of the range. Its low value could be due to the correlation between flow size and velocity. The lower the size, the lower the velocity and the lower the turbulent friction. This assumption is supported by the study of C. Chalk et al. [ACoLwL, p.575], where an experimental sand flow with 3kg mass had a turbulent friction value of only 20 ms$^2$. 
5. Practical application of avalanche dynamics

5.5 Conclusion

It has been successfully demonstrated how the Voellmy-Salm model can be applied to small-scale sand flows. The runout distances of twenty experimental sand flows for four different angles $\phi$ were recorded. This data was statistically processed using linear regression by the method of least squares [DA, chapter 9.3]. The resulting improved mean runout distances were than back-analyzed in order to calibrate the two Voellmy parameters $\xi$ and $\mu$. Using try-and-error by eye, a specific configuration with $\mu = 0.29$ and $\xi = 255$ was found. These values were than statistically validated using "The general case of least-square fitting" [DA, chapter 9.10] With these values, the Voellmy-Salm model closely calculated the runout distances which were observed in the experiment. As a conclusion, one can confidently assume that for inclination angles ranging from 35° to 50°, the Voellmy-Salm model is capable of quite accurately predicting the expected runout distances of small sand flows. My research question can thus be answered affirmatively. Of course, the validity of these findings, especially the values for $\mu$ and $\xi$, are strictly restricted to the chute, sand type, and sand mass utilized in this experiment. Nevertheless, the approximations of $\mu$ and $\xi$ and other knowledge gained from this practical work could be of use in future, related experiments.

5.6 Evaluation

One of the strengths of this practical application was the design of the chute. It allowed for easy comparison to the Voellmy-Salm model by keeping flow width constant for the total descent. Furthermore, its split into acceleration- and runout zone defined a clear point P. By slightly inclining the runout zone, the chute resembles more of a natural avalanche path. It was also long enough for the sand to reach a constant velocity. Additionally, the lid of the sand compartment limits the initial fracture height at 5 cm, which enables repeatability.

However, the manual lifting of the plate separating the sand compartment and acceleration zone might have been the source of minimal inaccuracy. When it is not lifted perfectly simultaneously at both ends, the runout distance on one side will be slightly larger than on the other. This could be improved by developing an automatic lifting device, e.g. by use of a spring.
In addition, there was minor sand leakage at the transition of the two zones. This could be reduced by better sealing.

The method used to validate the angle of \( \varphi \) turned out to be rather tedious. Both time efficiency and accuracy could be increased by using a digital angle measurement tool.

As already explained, a more precise method for measuring the flow velocities should be implemented in the future. This would enable to produce a more reliable approximation of the value for \( \xi \). Unfortunately, the required tools are quite costly. Yet, it might have been possible to also measure the frontal velocities by positioning the lid closer to the front of the sand compartment. It would also have been interesting to compare frontal and body velocities.

If this experiment was conducted for a second time, the derived values for \( \mu \) and \( \xi \) could be validated by keeping the angle \( \varphi \) constant and by changing the angle of the runout zone. Would the Voellmy-Salm model still correctly calculate the observed runout distances?

Furthermore, the moisture content in the sand could be increased to study its influence on the runout distance, velocity, and thus friction parameter values.
### 6. Bibliography

#### I. Internet resources

<table>
<thead>
<tr>
<th>Reference</th>
<th>Link</th>
<th>Last accessed</th>
</tr>
</thead>
<tbody>
<tr>
<td>L24</td>
<td><a href="https://www.mun.ca/biology/scarr/MGA2-06-Table1.html">https://www.mun.ca/biology/scarr/MGA2-06-Table1.html</a></td>
<td>18.12.2017</td>
</tr>
</tbody>
</table>

#### II. Studies

**Reference**

[B. Salm et al., 1990](http://www.slf.ch/ueber/geschichte/index_EN)


[C. Ancey et al., 2003](http://www.slf.ch/ueber/organisation/schnee_permafrost/schneephysik/index_EN)

C. Ancey, M. Meunier, D. Richard, Inverse problem in avalanche dynamics models, WATER RESOURCES RESEARCH, VOL. 39, NO. 4, published 22 April 2003,

[C. Ancey et al., 2004](http://www.slf.ch/ueber/organisation/warnung_praevention/risikomanagement/index_EN)

C. Ancey, M. Meunier, Estimating bulk rheological properties of flowing snow avalanches from field data, JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 109, published 24 January 2004,
6. Bibliography


III. Books

Reference


[HTL] Handbuch Technischer Lawinenschutz [german], Florian Rudolf-Miklau, Siegfried Sauermoser, Wilhelm Ernst & Son, 2011

[LRM] Landslide Risk Management, Oldrich Hungr, Robin Fell, Réjean Couture, Erik Eberhardt, CRC Press, 2005
7. Appendix

➢ Method 9.3

We fit a straight line to the four observed mean values of $s$.

The four means (denoted $s_j$), their standard error (denoted $\sigma_j$) and the angles $\varphi$ (denoted $\alpha_j$) are shown in Table 7. The angles were transformed into radian form for easier computability.

<table>
<thead>
<tr>
<th>$j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_j$ [rad]</td>
<td>0.61</td>
<td>0.70</td>
<td>0.79</td>
<td>0.87</td>
</tr>
<tr>
<td>$s_j$ [cm]</td>
<td>75.2</td>
<td>90.8</td>
<td>111.6</td>
<td>127</td>
</tr>
<tr>
<td>$\sigma_j$ [cm]</td>
<td>0.58</td>
<td>0.58</td>
<td>1.17</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 7: Values taken from Table 3

I follow the procedure explained in chapter 9.3 in [DA].

The straight line has equation

$$s_j = x_2 \cdot \alpha_j + x_1$$ #(3)

The unknown parameters are $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

where $x_1$ and $x_2$ stand for the line’s y-intercept and gradient respectively.

With $\alpha_0 = 0$, we obtain
\[
A = -\begin{pmatrix}
1 & \alpha_1 \\
1 & \alpha_2 \\
1 & \alpha_3 \\
1 & \alpha_4
\end{pmatrix} = -\begin{pmatrix}
1 & 0.61 \\
1 & 0.70 \\
1 & 0.79 \\
1 & 0.87
\end{pmatrix}, y = c = \begin{pmatrix}
75.2 \\
90.8 \\
111.6 \\
127
\end{pmatrix}.
\]

The matrices \( G_s \) and \( H \) are formed by inserting the last row of Table 7.

\[
G_s = \begin{pmatrix}
2.94 & 0 & 0 & 0 \\
0 & 2.94 & 0 & 0 \\
0 & 0 & 0.74 & 0 \\
0 & 0 & 0 & 1.11
\end{pmatrix}, \quad H = \begin{pmatrix}
1.71 & 0 & 0 & 0 \\
0 & 1.71 & 0 & 0 \\
0 & 0 & 0.86 & 0 \\
0 & 0 & 0 & 1.05
\end{pmatrix}.
\]

Hence, we obtain

\[
A' = -\begin{pmatrix}
1.71 & 1.05 \\
1.71 & 1.20 \\
0.86 & 0.67 \\
1.05 & 0.92
\end{pmatrix}, \quad c' = \begin{pmatrix}
128.97 \\
155.72 \\
95.70 \\
133.87
\end{pmatrix}, \quad A'^Tc' = -\begin{pmatrix}
711.4 \\
509.1
\end{pmatrix}, \quad (A'^TA')^{-1} = \begin{pmatrix}
8.01 & -11.29 \\
-11.29 & 16.17
\end{pmatrix}.
\]

Then, the solution becomes

\[
\bar{x} = \begin{pmatrix}
8.01 \\
-11.29
\end{pmatrix}^{-1} = \begin{pmatrix}
711.4 \\
509.1
\end{pmatrix} = \begin{pmatrix}
-47.48 \\
199.81
\end{pmatrix}.
\]

The covariance matrix of \( \bar{x} \) is

\[
C_{\bar{x}} = G_{\bar{x}}^{-1} = (A'^TA')^{-1} = \begin{pmatrix}
8.01 & -11.29 \\
-11.29 & 16.17
\end{pmatrix}.
\]

Their diagonal elements are the variances of \( \bar{x}_1 \) and \( \bar{x}_2 \) and their square roots are their errors.

\[
\Delta \bar{x}_1 = 2.83, \Delta \bar{x}_2 = 4.02.
\]

Applying equation 3 we obtain the improved means of \( s \) for each angle \( \varphi \).

\[
\bar{\eta} = \begin{pmatrix}
74.57 \\
92.0 \\
109.44 \\
126.88
\end{pmatrix}
\]

And the corresponding errors are the square roots of the diagonal elements of the covariance matrix \( C_{\bar{\eta}} \).

\[
\Delta \bar{\eta}_1 = 0.50, \Delta \bar{\eta}_2 = 0.36, \Delta \bar{\eta}_3 = 0.50, \Delta \bar{\eta}_4 = 0.79.
\]

The value of the minimum function, which is given by
$$M = \bar{\varepsilon}^T G y \bar{\varepsilon} = (y - \bar{\eta})^T G y (y - \bar{\eta}) = \left( \sum_{j=1}^{n} \frac{y - \bar{\eta}_j}{\sigma j} \right)^2$$

is

$$M = 8.89.$$  

With this result, I carry out a $\chi^2$-test on the goodness-of-fit of a straight line to our data. Since I started with $n = 4$ means and have determined $r = 2$ unknown parameters, the degree of freedom is $n - r = 2$. With a significance level of 0.01, we find the corresponding critical value in Picture 14 to be 9.21. As $M < 9.21$, there is thus no reason to reject the hypothesis of a straight line.

**TABLE 6-1** Critical Values of the $\chi^2$ Distribution

<table>
<thead>
<tr>
<th>df</th>
<th>0.999</th>
<th>0.9975</th>
<th>0.9</th>
<th>0.5</th>
<th>0.1</th>
<th>0.05</th>
<th>0.025</th>
<th>0.01</th>
<th>0.005</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.000</td>
<td>0.016</td>
<td>0.455</td>
<td>2.706</td>
<td>3.841</td>
<td>5.024</td>
<td>6.635</td>
<td>7.879</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.010</td>
<td>0.051</td>
<td>0.211</td>
<td>1.386</td>
<td>4.605</td>
<td>5.991</td>
<td>7.378</td>
<td>9.210</td>
<td>10.597</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.072</td>
<td>0.216</td>
<td>0.584</td>
<td>2.366</td>
<td>6.251</td>
<td>7.815</td>
<td>9.348</td>
<td>11.345</td>
<td>12.838</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0.207</td>
<td>0.484</td>
<td>1.064</td>
<td>3.357</td>
<td>7.779</td>
<td>9.488</td>
<td>11.143</td>
<td>13.277</td>
<td>14.860</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>0.412</td>
<td>0.831</td>
<td>1.610</td>
<td>4.351</td>
<td>9.236</td>
<td>11.070</td>
<td>12.832</td>
<td>15.086</td>
<td>16.750</td>
<td>5</td>
</tr>
</tbody>
</table>

Picture 14: Critical values of the $\chi^2$-distribution [L24]

➢ Method 9.10

I follow the process explained in chapter 9.10 in [DA].

The $r = 2$ unknown parameters are placed in a vector $\eta$. The measured values form a 4-vector $\eta$. The values actually measured $y = s$ differ from $\eta$ by the errors $\varepsilon$. The individual errors $\varepsilon_j$ ($j = 1, 2, 3, 4$) are assumed to be normally distributed with the null vector as the vector of expectation values and the covariance matrix $C_y = G_y^{-1}$. The $x$ and $\eta$ are related by $m = 4$ functions

$$f_k(x, \eta) = f_k(x, y - \varepsilon) = 0, \quad k = 1, 2, 3, 4.$$  

Further we assume that a first approximation $x_0$ of the unknown exists. As a first approximation of $\eta$ we use $\eta_0 = y$. The equations of the Voellmy-Salm model are the following:

$$v_0 = \sqrt{\xi d_0 (\sin \alpha - \mu \cos \alpha)}, \quad Q = B_0 d_0 v_0, \quad v_p = 3^{\frac{\xi}{B_p}} \left( \sin \alpha - \mu \cos \alpha \right), \quad d_s = \frac{Q}{B_p v_p} + \frac{v_p^2}{10g},$$

$$V^2 = d_s \xi (\mu \cos 5^\circ - \sin 5^\circ), \quad s = \frac{d_s \xi}{2g} \ln \left( 1 + \frac{v_p^2}{v^2} \right).$$
Further we assume that the function $f_k$ behaves linearly in the region around $(x_0, \eta_0)$. One develops $f_k$ up to the first order (formula (9.10.2)).

The values are taken from Table 3 and are equal to the ones in Table 7:

<table>
<thead>
<tr>
<th>$j$</th>
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<td>0.58</td>
<td>1.17</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 8: Original data taken from Table 3

Further we have:

$$f_k = \eta_k - \frac{d_x \xi}{2g} \ln \left(1 + \frac{v^2}{v^2}ight), \quad k = 1, \ldots, 4, \quad x = \left(\frac{\mu}{\xi}\right), \quad x_0 = \left(\frac{0.29}{255}\right), \quad \eta_0 = y = \left(\frac{75.2}{90.8}, \frac{111.6}{127}\right).$$

This process according to 9.10 in [DA] produces in a first round the following values:

$$\bar{x} = \left(\begin{array}{c} 0.2943 \\ 255.110 \end{array}\right), \quad \Delta \bar{x} = \left(\begin{array}{c} 0.0644 \\ 1.6995 \end{array}\right), \quad \bar{\eta} = \left(\begin{array}{c} 0.7578 \\ 0.9126 \\ 1.0813 \\ 1.2772 \end{array}\right), \quad \Delta \bar{\eta} = \left(\begin{array}{c} 0.2731 \\ 0.2731 \\ 0.2731 \\ 0.2731 \end{array}\right).$$

The minimum function $M$ goes as follows

$$M = (B \bar{x})^T G_B (B \bar{x}) = 0.00110081.$$

This value is too unrealistic to be used for a reliable $\chi^2$-test. And further rounds of method 9.10 are pointless.

A graphical $s$-$\alpha$-Diagramm shows that $s$ behaves virtually linearly in the angle interval of $35^\circ$ to $50^\circ$. This method is thus at least applicable in a first round.